ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 325
Introduction to Abstract Algebra I

1st Midterm

Nov 13, 2006

17:40-19:10

Surname : ____________________

Name : ____________________

ID # : ____________________

Department : ____________________

Section : ____________________

Instructor : ____________________

Signature : ____________________

- The exam consists of 5 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	

1. (20 pts.) Mark each of the following assertions True (**T**) or False (**F**). Justify your answer: give a proof or a counterexample.

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a) A finite nonempty subset of a group that is closed is a subgroup.

b) Every cyclic group is Abelian

c) Every Abelian group is cyclic.

d) Every cyclic group of order at least 3 has at least two generators.

e) In the group $(\mathbb{Z},+)$, the intersection $\langle 6\rangle \cap \langle 15\rangle \cap \langle 10\rangle$ is $\langle 60\rangle$.

2.

a) (10 pts.) Find the order of the group $U(30)$.

b) (10 pts.) Find the order of each element in the group $U(30)$ and determine whether $U(30)$ is cyclic or not.

3.

a) (10 pts.) Let $G = \langle a \rangle$ be a cyclic group of order n and let k be a positive integer. Find the order of the element a^k.

b) (10 pts.) Let $G = \mathbb{Z}_{80000}$. Using part a. find all elements of order 8 in G.

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4. Let G be group and let $x, y \in G$ be elements in G such that $|y| = 2$, $x \neq e$ and

$$yxy = x^2.$$

a) (5 pts.) Using the above equation show that $(yx)^2 = x^3$
b) (5 pts.) Using the above equation show that $yx = xyx^{-1}$ (*Hint:* multiply the above equation by x^{-1} from the right and use $|y| = 2$.)
c) (5 pts.) Prove that $|xyx^{-1}| = |y| = 2$ and find $|yx|$ using part b)
d) (5 pts.) Find $|x|$ using parts a) and c)

5.

a) (10 pts.) Let G be group, let $x \in G$. By the centralizer of x in G we mean the set

$$C_G(x) = \{a \in G \mid xa = ax\}$$

Prove that $C_G(x)$ is a subgroup of G.

b) (10 pts.) Let $x, y \in G$ and $xy \in C_G(x)$. Prove that $xy = yx$.

6. (15 pts.) Let $G = \mathbb{R}^* = \mathbb{R} - \{-1\}$ and define the operation $\star$ on G by $a \star b = a + b + ab$.

$$a \star b = a + b + ab$$

Prove that $(G, \star)$ is a group.